A Stochastic Humanitarian Inventory Control Model for Disaster Planning

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ABSTRACT

The impacts of disasters have been recently attracting increasing attention of researchers and policy makers. Nevertheless, there has been little consensus about how an efficient inventory management model can be developed for post-disaster conditions. Victims of a disaster are generally gathered into shelters during and after a severe disaster to ensure their security. Many of these evacuees do not have financial resources to leave the disaster area and to find food, drugs, or other necessities to survive. Hence, their vital needs should be supplied efficiently throughout the entire disaster and post-disaster periods. Without an adequate stock of goods, satisfying daily requirements of the disaster victims without disruptions might be problematic. To solve this problem, humanitarian inventory control models, which can provide adequate response to a disaster and/or humanitarian crisis, are needed. In this context, response represents “preparedness, planning, assessment, appeal, mobilization, procurement, transportation, warehousing, and distribution” (8). This paper is thus concerned with the development of a sub-problem of the general humanitarian supply-chain problem, namely, an efficient and quick-response humanitarian inventory management model which can determine the safety stock that will prevent disruptions at a minimal cost. We first mathematically formulate the humanitarian inventory management problem as a version of Hungarian Inventory Control Model. We then propose a solution to this time-dependent stochastic model using pLEPs algorithm. We conclude the paper by giving the single commodity case results and conducting a sensitivity analysis of our model vis-à-vis various model parameters that affect safe inventory levels.
INTRODUCTION

Natural disasters frequently happen in various parts of the world. For example, the Gulf States in the USA are extremely vulnerable to hurricanes such as the infamous Hurricane Katrina. Moreover, some countries like Turkey or Indonesia, experience severe earthquakes that can kill many people and seriously disrupt daily lives of people. In other places, there may be floods, wild fires, or other disasters that might require evacuation of people either before and/or after the specific disaster occurs. The important point is that they make all of us acutely aware of our vulnerabilities to disasters.

Therefore, we are mostly concerned with the effects of these disasters on the survivors. Can sick or hurt people have a chance to receive the medical care and supplies they need? Can people find enough food and adequate shelter so that they can survive for a relatively extended period of time after the disaster? With these important questions in mind, this study is concerned with the development of a realistic inventory management model.

We assume that people are gathered into shelters just before and/or after a disaster to ensure their security. Many of these people, as in the case of Hurricane Katrina, might not have financial resources to leave the disaster area and to find food, drugs, clothes or other basic needs. Others might want to stay close to their hopes and not want to travel too far mainly due to a desire to go back to their homes once the threat of the specific disaster is over. Therefore, the needs of these evacuees should be supplied efficiently throughout the entire pre and post-disaster period. The basic control concept, with the hierarchy of decisions, is illustrated in Figure 1.

![FIGURE 1 Basic Control and Hierarchy of Decisions](image)

To summarize, there are three basic issues to be considered during and/or after a disaster:
• Set up centers that stock inventories of supplies needed, receive and distribute them when they are shipped in.
• Install a state-of-the-art automated inventory management system that manages and distributes supplies to the evacuees.
• Implement an innovative stock rotation process that assures costly supplies not to expire in storage.

To obtain an approximate model for these issues, the delivery and consumption processes have to be carefully studied. The significant point is that the model has to allow the so-called delivery without disruption, by a prescribed high probability. That is, there should always be enough supply while delivery and consumption occur simultaneously. Moreover, both delivery and consumption should be modeled as stochastic processes. This paper aims to obtain such an efficient model by using the Hungarian Inventory Control model previously used to examine and remedy the problem of superfluously high inventory levels at Hungarian industrial plants (1).

Our proposed model attempts to determine the minimal safety stock level of an inventory, allowing a continuous consumption without disruption, by a probability \( \varepsilon \), where \( \varepsilon \) is a small number. With this information, it is aimed to response quickly and effectively to a complex emergency case due to a disaster.

LITERATURE REVIEW

Oh and Haghani (2) studied on the transportation of many different goods, such as food, clothing, and medical supplies to minimize the number of casualties and maximize the efficiency of the rescue operations. They formulated and solved a multi-commodity, multi-modal network flow model. Barbarosoglu and Arda (3) extended this model into a two-stage stochastic programming model where they include uncertainties due to estimation of resource needs of first-aid goods, vulnerability of resource suppliers and survivability of the routes in the disaster region.

Emergency planning models have been developed to focus on the delivery and storage of the products during the disasters. Barbarosoglu et al (4) focused on the use of helicopters for delivering supplies and saving victims during the disasters. Ozdamar et al. (5) presented a mathematical model for emergency logistics planning in disasters. Integration of two multi-commodity network flow problems, the model addressed a dynamic time-dependent transportation problem. Yang and Federgruen (6) also developed disaster planning models focusing on the selection of optimal suppliers by analyzing a planning model for a firm or public organization, covering uncertain demand for a given item by procuring supplies from multiple sources.

Apparently, the humanitarian supply chain models are the keys to solve the inventory control problem during the disasters. With this idea, Beamon and Kotleba (7) worked on humanitarian supply chain management, and developed a stochastic inventory control model determining optimal order quantities and reorder points for a long-term emergency relief response. Blanco and Goetzcel (8), reviewed the humanitarian supply chain studies, and provided valuable background information. They defined humanitarian supply chains as the supply chains that support response to a disaster and/or humanitarian crises. Response, here, represents preparedness and planning, assessment and appeal, mobilization, procurement, transportation, warehousing, and distribution to beneficiaries. Moreover, Akkihal (9) examined the impact of inventory pre-positioning on humanitarian
operations whereas Russell (10) described humanitarian relief supply chains for those specifically utilized in the 2004 South East Asia Earthquake and Tsunami relief effort and Davidson (11) tried to come up with the key performance indicators in humanitarian logistics.

Model Predictive Control (MPC) strategies serve a wide range of control methods that can be used in supply-chain management and inventory control problems. Braun et al. (12) showed that a MRC framework can handle the demand networks efficiently with a successful management. Rasku et al. (13) worked on an implementation of a better cost function in inventory control and supply-chain management problems whereas Dunbar and Desa (14) demonstrated an application of nonlinear model predictive control (NMPC) strategies.

In collaboration between Rensselaer Polytechnic Institute (RPI) and the University of Delaware, researchers are gathering perishable information and data about the basic features of the formal and the informal logistic systems supporting the recovery and the flows of critical and non-critical supplies to areas impacted by Hurricane Katrina. This project will identify the basic features of the supply chains delivering resources to the Gulf Coast, and document lessons learned, both positive and negative. They are concerned with the collection of response data in the weeks following the hurricane Katrina. This data is very important for models similar to the one proposed in this paper since it contains real life information that is not widely available (15).

In summary, mathematical modeling of inventory management systems used for the disasters has been recently receiving increasing attention in the literature. However, there is still a growing need for better and more realistic models. This research focuses on obtaining such an effective humanitarian inventory management model using Hungarian Inventory Control Model introduced by Prékopa (1).

**HUMANITARIAN INVENTORY MANAGEMENT PROBLEM**

**Parameters and Decision Variables**

\( T \): length of the finite time interval,
\( \tau \): time when an order is placed,
\( \varepsilon \): probability of disruption,
\( n \): number of delivery times in interval \( T \),
\( r \): number of items,
\( a \): space occupied by each item,
\( g \): cost of storage,
\( f \): cost of adjustment,
\( q^+ \): cost of surplus,
\( q^- \): cost of shortage,
\( S \): initial inventory level,
\( \delta \): minimum amount of stock supplied in a delivery time,
\( D \): total amount of stock/demand supplied,
\( m \): initial safety stock in interval \( T \),
\( s \): number of consumption times in interval \( T \),
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\( \gamma \) : minimal amount of goods consumed in a delivery time,
\( C \) : total amount of goods consumed,
\( M \) : total storage capacity,
\( x \) : sample from uniformly distributed population in the interval \([0, D - n\delta]\),
\( L \) : sample size of \( x \),
\( j \) : positive integers selected randomly up-to \( L \),
\( X \) : delivery process parameter,
\( y \) : sample from uniformly distributed population in the interval \([0, C - n\gamma]\),
\( N \) : sample size of \( y \),
\( k \) : positive integers selected randomly up-to \( N \),
\( Y \) : consumption process parameter,
\( W \) : random variable of the approximate multivariate normal distribution,
\( \mu \) : mean of the approximate distribution,
\( \sigma \) : standard deviation of the approximate distribution,
\( u \) : type of demand \( D \),
\( p \) : probabilities for each demand,
\( m_u \) : decision variable, additional safety stock to satisfy the probabilistic constraint,
\( M_l \) : decision variable, storage capacity of each item, \( l = 1...r \),
\( e \) : discrete variable approximating \( W \),
\( z \) : possible values of \( e \),
\( B \) : upper boundary of the interval \( z \) values are taken,
\( \zeta \) : prescribed small tolerance.

**Basic Assumptions**

The “humanitarian disaster inventory control” problem, as defined in this paper, is to find the amount of safety stock with a probability \( 1 - \epsilon \), so that delivery and consumption processes go on without disruption at minimum cost. For instance, if the value of \( \epsilon \), the probability of disruption, is 0.1, the aim is that disruption will not occur 90% of the time.

The following modeling assumptions can be made for a finite time interval \([0, T]\).

- Deliveries take place during an interval, according to some random process, rather than at a single time epoch.
- Deliveries take place at discrete times, fixed and designated by \( n \) which can be obtained from past history. The \( n \) delivery random times have joint probability distributions the same as that of \( n \) random points chosen independently from the interval \([\tau; \tau + T]\) according to a uniform distribution.
- The delivery and consumption processes are stochastically independent.
- In each delivery time \( n \), a minimal amount, \( \delta \), is delivered. If the total amount of delivery is \( D \), then there is also a random amount of delivery obtained by choosing a random sample of size \( n - 1 \), from a population uniformly distributed in the interval \([0, D - n\delta]\).
• The consumption process is defined similarly, with parameters $C$ as the total amount of consumption, $\gamma$ as the minimal amount of consumption, and $s$ as the number of consumption times.

**Stochastic Programming Type Model**

In the literature, these problems are modeled by using so-called “Hungarian Inventory Control Problem” as a stochastic programming model. Here, we will formulate the “disaster inventory control problem” as a “Hungarian Inventory Control Model” described by Prékopa (1). We will first mathematically describe a multi-commodity model in the context of humanitarian logistics.

Firstly, total time, $T$, is subdivided into $n$ periods. During this time, an amount $D$ is delivered to a shelter and an amount $C$ is consumed by the evacuees. Here, $D$ and $C$ are assumed to be known constants from the past data that can be obtained from the recent studies (9). Then, a model describing how the deliveries and consumptions in the subsequent time intervals take place is formulated. It is assumed that the delivery and consumption processes have the same number of delivery times, $n$.

In each time interval, a minimal amount equal to $\delta \geq 0$ is delivered. Then, a sample size of $L$ ($L \geq n$) is taken from the uniformly distributed population in the interval $[0, D-n\delta]$ and given by $x^*_1 \leq x^*_2 \leq \ldots \leq x^*_n$. Then, $n-1$ positive integers are taken as $j_1 < j_2 < \ldots < j_{n-1} \leq L$. The delivered quantities of the emergency commodity in the $n$ time intervals are assumed to be

$$\delta + x^*_1, \delta + x^*_2 - x^*_1, \ldots, \delta + x^*_{j_{n-1}} - x^*_{j_{n-2}},(D-n\delta) - x^*_{j_{n-1}}.$$

The model for the consumption is similar. A sample size of $N$ is taken from the population uniformly distributed in the interval $[0, C-n\gamma]$, and designated by $y^*_1 \leq y^*_2 \leq \ldots \leq y^*_n$. Then, $n-1$ positive integers are taken as $k_1 < k_2 < \ldots < k_{n-1} \leq N$. The delivered quantities are assumed to be

$$\gamma + y^*_1, \gamma + y^*_2 - y^*_1, \ldots, \gamma + y^*_{k_{n-1}} - y^*_{k_{n-2}},\gamma + (C-n\gamma) - y^*_{k_{n-1}}.$$

Assuming that the delivery and consumption processes are independent, we let

$$X_1 = x^*_1, X_2 = x^*_2 - x^*_1, \ldots, X_{n-1} = x^*_{j_{n-1}} - x^*_{j_{n-2}}, X_n = (D-n\delta) - x^*_{j_{n-1}},$$

$$Y_1 = y^*_1, Y_2 = y^*_2 - y^*_1, \ldots, Y_{n-1} = y^*_{k_{n-1}} - y^*_{k_{n-2}}, Y_n = (C-n\gamma) - y^*_{k_{n-1}}.$$

Let $S$ denote the initial safety stock of commodities. Another assumption is that we will always have a safety stock, shown as $S + D \geq C$. Then, the condition of no disruption is formulated as follows:

$$S + \delta + X_1 \geq \gamma + Y_1,$$

$$S + 2\delta + X_1 + X_2 \geq 2\gamma + Y_1 + Y_2,$$

$$\vdots$$

$$S + (n-1)\delta + X_1 + X_2 + \ldots + X_{n-1} \geq (n-1)\gamma + Y_1 + Y_2 + \ldots + Y_{n-1},$$

$$S + n\delta + X_1 + X_2 + \ldots + X_n \geq n\gamma + Y_1 + Y_2 + \ldots + Y_n.$$

The last inequality $S + D \geq C$ is simply removed since it is aimed to have a model without disruption.
Thus, a stochastic programming model serves for the determination of the initial safety stock levels. Assume that delivery and consumption processes are independent and there will be a superscript \( l \) for each item. Then, \( P(S^l) \) is defined as the probabilities that all inequalities above are satisfied and approximated by the joint probability distribution of the random variables given as

\[
W^{(l)}_i = i\gamma^{(l)} + Y_1^{(l)} + \ldots + Y_n^{(l)} - i\delta^{(l)} - X_1^{(l)} - \ldots - X_u^{(l)}, \quad l = 1, \ldots, r; i = 1, \ldots, n.
\]

The joint distribution of these random variables is approximated as a multivariate normal distribution. The expectation and variance of the random variable \( W^{(l)}_i \) can be calculated as follows (1).

\[
\begin{align*}
\mu_i^{(l)} &= i\gamma^{(l)} + (C^{(l)} - n\gamma^{(l)})\left( \frac{k_i^{(l)}}{N^{(l)}} + 1 \right) - (D^{(l)} - n\delta^{(l)})\left( \frac{j_i^{(l)}}{L^{(l)}} + 1 \right), \\
(\sigma_i^{(l)})^2 &= (C^{(l)} - n\gamma^{(l)})^2\left( \frac{k_i^{(l)}}{N^{(l)}} + 1 \right)^2\left( \frac{1}{N^{(l)} + 2} \right) + (D^{(l)} - n\delta^{(l)})^2\left( \frac{j_i^{(l)}}{L^{(l)}} + 1 \right)^2\left( \frac{1}{L^{(l)} + 2} \right).
\end{align*}
\]

Then, the overall multi-item two-stage stochastic programming problem becomes

\[
\begin{align*}
\min \left\{ T \sum_{i=1}^r g^{(i)}(M^{(i)}) + \frac{1}{T} \sum_{u \in U} p_u \left[ f^{(i)}(m_u^{(i)}) + \sum_{i=1}^n q_i^{(i)} m_u^{(i)} + \sum_{i=1}^n q_i^{(i)} - q_i^{(i)} \int_{\mu^{(i)} - \sigma^{(i)}}^{\infty} (1 - \Phi(z))dz \right] \right\} \\
\text{Subject to} \\
\prod_{i=1}^r \Phi\left( \frac{m_u^{(i)} + m_u^{(i)} - \mu_u^{(i)}}{\sigma_u^{(i)}} \right), \quad i = 1, \ldots, n - 1; R_i) \geq 1 - \varepsilon \\
m^{(i)} + m_u^{(i)} \leq M^{(i)}, m_u^{(i)} \geq 0, u \in U, l = 1, \ldots, r \\
\sum_{i=1}^r a_i M^{(i)} \leq M.
\end{align*}
\]

The demand is an \( r \)-component random vector \( D = (D^{(1)}, \ldots, D^{(r)}) \) on discrete supports \( \{D_u, u \in U\} \), where \( U \) is a finite set. The corresponding probabilities for each \( \{D_u, u \in U\} \) are given by \( p_u \). For this model, the demand is taken as equal to the consumption.

In the two-stage problem, there are first and second stage decision variables; a subscript \( u \) is given to each second stage variable. The first stage variables are \( M^{(i)}, l = 1, \ldots, r \) and the second stage variables are \( m_u^{(i)} \geq 0, u \in U, l = 1, \ldots, r \). In the first stage, we decide the values \( M^{(i)}, l = 1, \ldots, r \), the storage capacities corresponding to the \( r \) items. The \( g^{(i)}(x), l = 1, \ldots, r \) are the convex cost functions of storage capacities \( M^{(i)}, l = 1, \ldots, r \). The second stage problem comes up after the demand values \( D = (D^{(1)}, \ldots, D^{(r)}) \) are observed. We prescribe that no disruption occurs in any of the \( r \) consumptions in the time intervals \( (kT + \tau; (k + 1)T + \tau), l = 1, \ldots, r \), with probability \( 1 - \varepsilon \). This parameter, \( \varepsilon \), represents the probability of disruption due to the unavailability of the required item or disruptions in the transportation system, etc. The optimal values of the second stage variables \( m_u^{(i)} \geq 0, u \in U, l = 1, \ldots, r \), are the adjustment values of the safety
stocks to make the probabilistic constraints feasible. If at time \( kT + \tau \), the safety stock levels are \( m^{(l)}_u \geq 0, u \in U, l = 1, ..., r \), the new stock levels calculated are \( m^{(l)} + m^{(l)}_u \geq 0, u \in U, l = 1, ..., r \). Here, the adjustments incur some costs; the adjustment cost function of consumption item \( l \) is denoted by \( f^{(l)}(x), l = 1, ..., r \).

**Constraints**

It is significant to consider the probabilistic nature of the consumption and delivery processes given the highly stochasticities of the problem domain dealing with events that will occur just before and after a natural or man-made disaster. Unlike relatively predictable conditions of our daily lives, disaster conditions are highly stochastic. Thus, our model should take these stochasticities into account in terms of probabilistic constraints. There are two types of constraints in the model, the probabilistic constraints, and the capacity constraints.

For the probabilistic constraints, we have to concentrate on the random variable \( W^{(l)}_i \). \( W^{(l)}_i \) simply represent the “Consumption–Delivery” for any time step \( i = 1, ..., n \) so that our probabilistic constraint is

\[
P(W^{(l)}_i \leq m^{(l)} + m^{(l)}_u, i = 1, ..., n - 1; R_i) \geq 1 - \varepsilon.
\]

It is clear that sum of initial stocks and deliveries has to be greater than or equal to the consumption for any time step \( i = 1, ..., n \). By replacing \( W^{(l)}_i \) with their expectations and variances, this constraint is turned into

\[
\prod_{i=1}^{n} \Phi\left(\frac{m^{(l)} + m^{(l)}_u - \mu^{(l)}_u}{\sigma^{(l)}_{u}}, i = 1, ..., n - 1; R_i\right) \geq 1 - \varepsilon.
\]

The other constraints are the capacity constraints. At any time, initial safety stock plus the optimal additional stock must be smaller than the storage capacity for that item, and the sum of storage capacities for each item must be smaller than the overall capacity.

**Objective Function**

The objective cost function is the sum of individual costs listed below:

- **Cost of Storage:** It is obvious that there is a cost for storing each commodity. In case of disaster operations, it is important to consider storage costs since the occurrence of a disaster is not known a priori.
- **Cost of Surplus:** This is incurred if there is more inventory than demand. It can be modeled as a fixed cost or as a step function that allows very low or no cost for a certain surplus level and then a steep increase for higher levels of surplus.
- **Cost of Shortage:** This cost is incurred if there is not sufficient inventory to satisfy the demand of the evacuees. This is the most important cost component, as clearly seen in the case Katrina and other recent disasters, since shortage of vital supplies can even cause loss of life.
- **Cost of Adjustment:** This cost is incurred by the nature of the two-stage model. Suppose we have an initial amount of safety stock, but to satisfy the probability constraint, we need more. This adjustment can be due to the unexpected factors such as the strength of a hurricane or an earthquake, increased number of people
who are affected and need help, etc. Of course, this has to be penalized. It can be chosen as a linear function of the additional stock.

**Proposed Solution Approach**

To obtain the solution of the aforementioned convex nonlinear programming problem, the concept of p-level efficient points (pLEPs) (16), is used. With this method, an approximate (discretized) version of the main problem can be obtained. The multi-item problem requires the calculation of the multiplication of normal cumulative distribution functions. Therefore, before applying pLEPs method, the continuous distribution functions have to be converted into discrete distributions to apply pLEPs as shown in (17). Therefore we approximate the random variable $W$ by a discrete variable $e$ with possible values $z_1 < z_2 < ... < z_N$ where the distribution function is as follow:

$$F(W) = \begin{cases} F(e), & i = 1, ..., N - 1 \\ 1, & i = N \end{cases}$$

(4)

The values $z_1 < z_2 < ... < z_N$ are chosen to be equidistant on some interval $[0, B]$ where $F(B) = 1 - \zeta$ for a prescribed small tolerance $\zeta$, such as 0.01.

**Humanitarian Inventory Control Model for the Single Commodity Case**

Our main goal is to find the optimal amount of initial stock so that no disruption occurs during the delivery and consumption processes, with a chosen probability of disruption. In this section, we will present the single-commodity case of the problem. This case enables us to study the impact of model parameters such as the probabilistic consumption and delivery factors, initial stock levels, etc. without loss of generality. The single commodity mathematical formulation is as follows:

$$\min \left\{ \frac{1}{T} \sum_{u \in U} p_u \left[ f(m_u) + \sum_{i=1}^{n} q_i m_u + \sum_{i=1}^{n} (q_i^* + q_i^-) \int_{m_u}^{\infty} (1 - \Phi(\frac{z - \mu_{iu}}{\sigma_{iu}}))dz \right] \right\}$$

subject to

$$\Phi(\frac{m + m_u - \mu_{iu}}{\sigma_{iu}}, i = 1, ..., n - 1; R_i) \geq 1 - \epsilon$$

$$m + m_u \leq M, m_u \geq 0, u \in U$$

Accurate statistical distributions of delivery and consumption processes are needed to solve and study the above model. In this paper, due to lack of such realistic data, pre-determined discretized normal distributions are adopted.

The problem can be solved for a range of demand values allowing us to observe the changes in safety stock values for different consumption amounts. During a post-disaster condition, the requirements of people can drastically change. Therefore, sometimes, it is logical to choose an initial safety stock that may increase the inventory cost, but can prevent disruption due to the stochastic demand changes. Analysis with different demands of evacuees will enable us to assess the overall picture more clearly.

The objective function includes these demands multiplied by their corresponding probabilities. This allows us to calculate the total cost where the highest and lowest demands have the lowest probabilities according to a pre-determined discretized normal distribution. For instance, the cost for a severe disaster demand may be higher than others.
due to additional safety stocks required, however, the probability associated with this high demand will be lower than other demands closer to the mean (normal distribution assumption). This self controlling mechanism gives the analysts the chance to determine the safety stock values more accurately. Moreover, as the shortage of commodities is our main concern, the penalty cost of shortage is chosen more than the one due to surplus.

The first constraint aforementioned is the so-called probabilistic constraint. The mean and variance of the distribution and the probability of disruption are used to calculate the safety stock values and the changes in safety stocks for different demands. That is, during a severe disaster, the demand values can change dramatically in such a way that the possibility of disruption will increase. The second constraint is the capacity constraint which ensures that the total safety stock never exceeds the total storage capacity of goods like food or medicine.

In the next section, the results and sensitivity analysis of the single item case of our application to disaster supply-chain management problem are given.

RESULTS & SENSITIVITY ANALYSIS OF THE SINGLE ITEM CASE

A base condition is established, and a sensitivity analysis is conducted by changing the model parameters to observe the behavior of the proposed model. For the base case, the following values are chosen.

- $\varepsilon$ is selected as 0.1 so that the probability $1 - \varepsilon$ in the constraint is equal to 0.9.
- The number of deliveries in the time interval, say a month, is chosen as $n = 10$ and the parameters $\delta$ and $\gamma$ are taken to be 0.2 and 0.25 units, respectively.
- The amount of initial safety stock, $m$, is 2 units.
- The surplus and the shortage costs are chosen as $q^+ = 0.1/\text{unit}$, and $q^- = 100/\text{unit}$.
- The cost of adjustment function is selected as $f(x) = 2x$.
- The cost of storage for each good is $1/\text{unit}$.
- Total storage capacity is considered to be 20 units.
- The space occupied by each item is 2 units.
- The delivery (demand) and consumption values are taken as $D = [3.2, 3.4, 3.6, 3.8, 4.0, 4.2]$, and $C = [5.2, 5.4, 5.6, 5.8, 6.0, 6.2]$, respectively.

The probability values of the discrete supports of consumption and delivery values are taken as $p = [0.13, 0.17, 0.20, 0.20, 0.17, 0.13]$.

The results obtained for the base case are given in Table 1. To satisfy the needs of the impacted people 90% of the time, initial stock has to be more than 50% of the total expected consumption. That is, having more than half of the commodities in inventory before the disaster, we will prevent disruption 90% of the time. Of course, this result is only for the base case. By changing the parameters, it is possible to have higher additional safety stock values and costs. For example, it may not be possible to make 10 deliveries in a given period of time, so additional amount of safety stock increases, incurring higher costs.
TABLE 1 Results for the base case (m=2)

<table>
<thead>
<tr>
<th>Demand</th>
<th>( u=1 )</th>
<th>( u=2 )</th>
<th>( u=3 )</th>
<th>( u=4 )</th>
<th>( u=5 )</th>
<th>( u=6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Stock</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Optimal Additional Stock</td>
<td>0.86</td>
<td>0.95</td>
<td>1.04</td>
<td>1.13</td>
<td>1.21</td>
<td>1.31</td>
</tr>
<tr>
<td>Total Initial Stock</td>
<td>2.86</td>
<td>2.95</td>
<td>3.04</td>
<td>3.13</td>
<td>3.21</td>
<td>3.31</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Consumption</td>
<td>55%</td>
<td>55%</td>
<td>54%</td>
<td>54%</td>
<td>54%</td>
<td>53%</td>
</tr>
<tr>
<td>Total Expected Consumption</td>
<td>5.20</td>
<td>5.40</td>
<td>5.60</td>
<td>5.80</td>
<td>6.00</td>
<td>6.20</td>
</tr>
<tr>
<td>The Optimal Value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.99</td>
<td></td>
</tr>
</tbody>
</table>

In the following sensitivity analysis results, values of every parameter are kept same as in the base case, except mentioned otherwise. The additional safety stock value changes are given for three demand values. The demand values are in an ascending order, demand type 1 represents the lowest demand, whereas demand type 6 is the highest.

**Changes in the Probabilities of the Disruption**

Epsilon value, \( \varepsilon \), is changed from 0.05 to 0.45 to see the behavior of the proposed model when the probability of not serving the evacuees, is increased. In other words, the probability of satisfying the requirements of all the evacuees is decreased from 95% to 55% of the time. The impact of all the changes is shown in Figure 2 where the additional safety stock decreases as the probability bound is relaxed. In this case, the total cost reaches a value including only the cost of initial stock as there is no additional safety stock required in the end. That is, the cost incurred in the end is only due to the storage cost of initial safety stock. As the cost of storage for one stock is chosen as 1, the resulting cost is the same as the initial safety stock amount. However, it is possible to increase the importance of the initial safety stock by changing the unit cost of storage. Occupied spaces for safety stocks also decrease as they have a positive correlation with the safety stocks.

Another important issue is to study the model for different distributions. By changing the mean and variance of the normal distribution used in the base model, we can have a better picture of the impact of the probabilistic occurrences. These changes can be thought as a way of representing the severity of the disaster. For a category 5 hurricane, the requirements of the evacuees can be very high, or the daily demand for medicine or shelter can have very high variances depending upon the time-dependent impact of the disaster. To get an insight of the above situation, two different distributions are analyzed. The severe hurricane having twice the demand of the base case requires more safety stock leading to higher costs.
Ozbay K., Ozguven E. E.

(a)

(b)
FIGURE 2 Changes in Additional Safety Stock Values (Figure 2a), Occupied Spaces of Safety Stocks (Figure 2b), Total Cost for the Base Case (Figure 2c) and Total Costs for Different Distributions (Figure 2d) for Different Probabilities

(c)

(d)
Changes in Initial Safety Stock

The main purpose of this research is to find the safety stock value that minimizes the cost while preventing disruption, for a pre-determined probability distribution. Therefore, it is important to observe the behaviour of additional stocks denoted as $m_u$, as the initial safety stock, $m$, is increased. As expected, $m_u$ decreases as $m$ increases. Then, the total cost tends to reach the one including only the storage cost of initial safety stock as no costs come from the additional safety stock values obtained to satisfy the probabilistic constraint given in Equation (3). The behavior of our model in response to an increase in initial safety stock can be seen in Figure 3. The decision maker has to determine the initial safety stock considering the total cost and the possibility of higher demand levels. For a major earthquake, the initial safety stocks can be selected as higher than the optimal values, to be on the safe side. Of course, this analysis is done for a single commodity and it has to be repeated for other commodities since the importance and shelf life of commodities are different.
FIGURE 3 Changes in Additional Safety Stock Values (Figure 3a) and Total Cost (Figure 3b) versus Initial Stocks

Changes in the Number of Deliveries

The number of deliveries during the chosen interval, such as a month, has the utmost importance for our model. In case of disasters, the changes in the number of deliveries mainly due to increased demand, reduced supply and possible disruptions in the transportation system are expected. Hence, it is significant to see how the model behaves due to changes in the number of deliveries. The results can be seen in Figure 4. As the amount of deliveries is increased, the additional amount of safety stock values tends to decrease. This is a logical behaviour since the increase in the initial safety stock leads to lower additional stock values to satisfy the probability constraint. For instance, when the number of deliveries reaches 22 for the whole time interval in this case, there is no need for additional safety stock. The initial safety stock is large enough to satisfy the demand of evacuees. Hence, total cost incurred decreases with lower levels of additional safety stocks. If the system is highly stochastic, then the probabilistic constraints are satisfied only if additional safety stock is high, too. Of course, this will impose higher costs. On the other hand, if the initial safety stock is too low, then the problem will become infeasible. Thus, our model achieves a balance between satisfying probability constraints and keeping the cost low.

The realization of the increase in the number of deliveries may not be possible due to the lack of trucks, personnel and the commodity itself, or due to transportation related problems such as loss of vital connectors or links. Therefore, by using the results of this analysis, it is possible to choose the safety stock value corresponding to the number of deliveries that can be comfortably achieved in the aftermath of a major extreme event such as a hurricane or an earthquake.
Ozbay K., Ozguven E. E.

(a)

(b)

The additional safety stock becomes zero at this point.

Cost due to initial safety stock is left only.
FIGURE 4 Changes in Additional Safety Stock Values (Figure 4a), Total Cost for the Base Case (Figure 4b) and Total Cost for Two Different Distributions (Figure 4c) versus Changes in the Number of Deliveries

The behavior of the system is observed for two different distributions as shown in Figure 4c. The main question is the following: “How does total cost change according to different demands and number of deliveries?” The answer is that there is a significant amount of difference in safety stock as the severity of the disaster increases where more safety stock results in substantial amount of increase in total cost.

Changes in the Amount of Consumption

Changing the consumption values depending on the severeness of the disaster, it is possible to figure out the changes in the initial safety stock values. In other words, the mean and variance of our random variable $W$ is increased to model the impact of the severeness of the disaster as far as the demand and consumption values are concerned. Therefore, the consumption values are increased to $C = [7.2, 7.4, 7.6, 7.8, 8.0, 8.2]$, and the amount of consumption in one period, $\gamma$, is taken as 0.45. The results are shown in Table 2. When the expected consumption is increased, the initial stock level chosen as 2 units appears to be quite low. Therefore, the additional safety stock levels are higher than expected leading to higher percentages of initial stock compared with the consumption values. The analysis is conducted one more time increasing $m$ by 2 units. Apparently, it is logical to have a higher initial safety stock for this case, as the cost associated becomes lower.
TABLE 2 Results for the increased consumption case

<table>
<thead>
<tr>
<th>Demand</th>
<th>u=1</th>
<th>u=2</th>
<th>u=3</th>
<th>u=4</th>
<th>u=5</th>
<th>u=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Stock</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Optimal Additional Stock</td>
<td>2.86</td>
<td>2.95</td>
<td>3.04</td>
<td>3.13</td>
<td>3.22</td>
<td>3.31</td>
</tr>
<tr>
<td>Total Initial Stock</td>
<td>4.86</td>
<td>4.95</td>
<td>5.04</td>
<td>5.13</td>
<td>5.22</td>
<td>5.31</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Consumption</td>
<td>68%</td>
<td>67%</td>
<td>66%</td>
<td>66%</td>
<td>65%</td>
<td>65%</td>
</tr>
<tr>
<td>Total Expected Consumption</td>
<td>7.20</td>
<td>7.40</td>
<td>7.60</td>
<td>7.80</td>
<td>8.00</td>
<td>8.20</td>
</tr>
</tbody>
</table>

The Optimal Value 15.79

Results for the increased consumption case (m=4)

<table>
<thead>
<tr>
<th>Demand</th>
<th>u=1</th>
<th>u=2</th>
<th>u=3</th>
<th>u=4</th>
<th>u=5</th>
<th>u=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Stock</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>Optimal Additional Stock</td>
<td>0.86</td>
<td>0.95</td>
<td>1.04</td>
<td>1.13</td>
<td>1.22</td>
<td>1.31</td>
</tr>
<tr>
<td>Total Initial Stock</td>
<td>4.86</td>
<td>4.95</td>
<td>5.04</td>
<td>5.13</td>
<td>5.22</td>
<td>5.31</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Consumption</td>
<td>68%</td>
<td>67%</td>
<td>66%</td>
<td>66%</td>
<td>65%</td>
<td>65%</td>
</tr>
<tr>
<td>Total Expected Consumption</td>
<td>7.20</td>
<td>7.40</td>
<td>7.60</td>
<td>7.80</td>
<td>8.00</td>
<td>8.20</td>
</tr>
<tr>
<td>The Optimal Value</td>
<td>8.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5 shows the changes in additional safety stock and cost due to the change in average consumption amount, starting from the case where delivery equals the consumption. As the consumption of the evacuees increases, the safety stock and the cost also increase.

(a)
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FIGURE 5 Changes in Additional Safety Stock Values (Figure 5a) and Total Cost (Figure 5b) versus Changes in Average Consumption Amounts

CONCLUSIONS AND FUTURE RESEARCH

The major goal of this paper is to develop a realistic model of the time-dependent inventory planning and management problem that can be used for the development of efficient pre and post-disaster plans. The proposed model attempts to determine the minimum safety stock so that the consumption of these stocked goods can occur without disruption for a given probability. That is, when evacuees are gathered into a major shelter such as the super dome in New Orleans, they must have enough supply of their vital needs. Moreover, all of these needs must be satisfied during this time period without any disruption, according to certain consumption and supply probabilities. The Hungarian Inventory Control Model (1) is used as the basic approach to model this important yet challenging problem.

A solution procedure based on the concept of p-level efficient points (pLEPs) is also proposed and various sensitivity studies are conducted for the single commodity version of the developed model. Furthermore, the behavior of the system in response to the changes to its most important decision parameters is studied. The changes in number of deliveries, probability of disruption and initial safety stock are all found to create a need for additional safety stock levels. As the demand of evacuees for a given commodity such as medicine increases, the corresponding safety stock values and incurred cost increase, too. Another example is that the more number of deliveries in a given interval is, the less additional stock values are required. Thus, if the transportation and the supply systems are robust enough to support higher level of deliveries, the authorities can afford to maintain lower levels of safety stocks. This will reduce the cost of emergency
preparedness incurred by various agencies without compromising the wellness of evacuees.

The results of this sensitivity analysis are encouraging in the sense that this model can be applied to study and better understand the so-called “humanitarian inventory control problem”. Moreover, the more complex systems that consist of multi-commodities can also be studied using the general model given in Equation (3). The multi-supplier case is also another important topic for future study.

The transportation of any good should have a significant effect, therefore additional cost items, the transportation and/or procurement costs of each item can be added to the objective function of our proposed model. The most important issue is obviously testing our model using real life data. This type of study will show whether or not the model works properly and efficiently. Hence, data validation and verification have to be performed using the data being collected in real life case studies that are currently underway (9).

REFERENCES
Ozbay K., Ozguven E. E.


